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Legal Positivism and Russell's Paradox

David Gray Carlson

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LEGAL POSITIVISM AND RUSSELL'S PARADOX

DAVID GRAY CARLSON*

ABSTRACT

In this Article, I argue that legal positivism is subject to the same paradox as was engendered by Frege's set theory–a paradox that has come to be known as Russell's Paradox. Basically, Frege tried to define what a set is. Russell showed that, because of self-reference, any attempt to define the word "set" led to formal condition. I argue that Russell's analysis can be applied to legal positivism, if "legal positivism" is defined to mean that a complete and closed rule of recognition for law is a logical possibility. I also argue that, to the extent legal positivism claims that law is not necessarily related to morality, then legal positivism is committed to the claim that there is a determinate rule of recognition. Only then can one be sure that a given law is purely a legal rule and not a moral rule. Because of the paradox, it is not logically coherent to divorce law and morality entirely, because no final and complete rule of recognition can possibly exist.

TABLE OF CONTENTS

INTRODUCTION 258
I. POSITIVISM AS A CLOSED SYSTEM 260
- *A. Kelsen* 261
- *B. Hart* 262
- *C. Raz* 268
- *D. Coleman* 272
- *E. Rawls* 274
- *F. Marmor* 275
- *G. Waluchow* 276
- *H. Schauer* 276

II. RUSSELL'S PARADOX 277
- *A. The Basic Demonstration* 277
- *B. The Axiom Scheme of Separation* 281
- *C. Responses* 283

* Professor of Law, The Benjamin N. Cardozo School of Law, Yeshiva University.

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III. RAZ'S PARADOX 286
CONCLUSION 288

INTRODUCTION

> But we cannot specify in advance what empirical criteria are permissible in the application [of] the formal concepts . . . . We cannot impose any limitations in advance on the possible formal analogies [i.e., models] which may permit the use . . . of the formal concepts in ways antecedently undreamt of by us. There must be conditions, directly or indirectly related to . . . intuition . . . for any employment . . . of the formal concepts . . . . But those conditions are not limited in advance by the scope of our actual knowledge and experience. . . . To suppose this would be a kind of restrictive dogmatism as unjustified in its way as the inflated dogmatism which pretends to a knowledge transcending experience. The latter makes an unjustifiable *a priori* claim to expand knowledge beyond experience. The former would make an equally unjustifiable *a priori* claim to restrict reality within the bounds of the kind of experience we in fact have.
>
> P.F. Strawson[1]

In 1902, Bertrand Russell proved that the set theory of the nineteenth century (as propounded by Gottlob Frege) was contradictory. Its defect was that it was *intensive* in nature. Frege had endeavored to set forth the rules of recognition by which every empirical example of "set" could be identified. These rules of recognition (or axioms) preceded the incidents of "sethood." The whole preceded the parts, and the whole was a *rule*.[2] A set, in this theory, could be precisely defined. Frege could describe the set of all sets.

But if this was so, one allowable set in Frege's system was the set of all sets that do not belong to themselves. This self-referential set fuels Russell's Paradox—the downfall of intensive set theory. If such a set

1. P.F. STRAWSON, THE BOUNDS OF SENSE: AN ESSAY ON KANT'S *CRITIQUE OF PURE REASON* 166–67 (1966). This volume thanks H.L.A. Hart for reading every sentence of the pre-publication manuscript. And so this is a paragraph with which Hart was quite familiar, though, in his *Concept of Law*, he would largely ignore its good advice.

2. Quine describes intension as follows: "for any condition you can formulate, there is a class whose members are the things meeting the condition. This principle is not easily given up." W.V. QUINE, THE WAYS OF PARADOX AND OTHER ESSAYS 11 (1966).

belongs to itself, it does not belong to itself. If it does not belong to itself, it belongs to itself. This constitutes a formal contradiction.[3] After Russell, set theory was reformulated to avoid this contradiction. The solution was to outlaw any reference to *totality*—to the set of all sets.[4]

Russell's Paradox obliged set theory to abandon intension in favor of *extension*. Extensive set theory refuses to say what sets are.[5] It refuses to define the set of all sets and therefore never encounters the set of all sets that do not belong to themselves. Sets are subjectively intuited by the observer; they are not established by pre-existing rules of recognition. In post-Russellian set theory, the parts precede the whole. The parts cannot be reduced to a rule of recognition. Ironically, the consequence of Russell's discovery—now universally accepted as correct—is that set theory has no theory of the set. It cannot say what a set is. Any attempt to theorize sets universally results in contradiction.

Legal positivism asserts that law is a set that does not belong to itself. For legal positivism, law is the set of all primary and secondary rules selected according to conventional rules of recognition. Whereas every individuated law is conventional, the fact that law *is* this set is not and cannot be conventional. Therefore, legal positivism is a set that does not belong to itself and so suffers from contradiction.

Over 100 years after Russell's Paradox, legal positivism still claims that intension (i.e., "meaning")[6] is either possible (soft positivism) or absolutely necessary (hard positivism). Accordingly, it is possible to apply Russell's Paradox to legal positivism in order to expose its contradictory nature. From this it follows that legal philosophy must give up on intension—on positivism itself—and be content with extension only. In an extensive jurisprudence, law cannot be reduced to a closed set of rules by which law can be recognized. In effect, we cannot say in advance what law is. The set of primary rules cannot be reduced to a determinate rule of recognition. We can only compile incomplete sets of individual laws.

Positivism usually claims that law, on the one hand, and morality, on the other, are not necessarily connected. This claim, however, depends upon the possibility that every primary rule of law has its origin in human legislation, rather than in reason. It is thought possible (or perhaps

3. In set theory terms, the set of all sets that do not belong to themselves asserts $(x \in x)$ & $-(x \in x)$, where $\in$ stands for "belongs to."

4. THOMAS JECH, SET THEORY 4 (3d ed. 2000) ("The set of all sets does not exist").

5. MARY TILES, THE PHILOSOPHY OF SET THEORY 175 (1989).

6. Hilary Putnam, *The Meaning of "Meaning," in* MIND, LANGUAGE AND REALITY 215, 219 (Hilary Putnam ed., 1975).

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necessary) to close off the set of primary rules from any rule with its origin in universal moral principle. Only if this is true can we affirm that law and morality are (possibly or necessarily) disjoint sets. Yet, if Russell's Paradox applies to jurisprudence as well as to set theory, this claim must be viewed as formally *impossible*, in which case positivism must be abandoned as a failed theory.

The purpose of this paper is to describe modern Zermelo-Fränkel set theory and apply it to positivist jurisprudence. The result is the invalidation of positivist jurisprudence, to the extent that it claims that law is reducible to the rules of its recognition. But first, I review the positivist literature to determine whether those who identify themselves as positivists in fact purport to "close the system" by reducing law to the rules of its recognition. Many prominent positivists are equivocal. As to these, the question arises whether they are entitled to be called positivists at all. For the equivocal theorist, law cannot be vouchsafed as having an origin solely in human legislation. There are, however, self-described positivists who do insist on the possibility of an intensive jurisprudence. These latter positivists are guilty of propagating an invalid and contradictory theory.

After reviewing various empirical positivisms, I will proceed to demonstrate that Russell's Paradox can be applied to positivist jurisprudence, understood as a purely intensive enterprise. I will try to anticipate some objections to this application. Finally, I will examine a claim by the arch-positivist Joseph Raz that positivism is indeed guilty of paradox. I will show that Raz's claim is *not* a version of Russell's Paradox. Indeed, Raz's demonstration is merely a logical mistake of the type known as *quaternio terminorum*. Positivism is therefore innocent of Raz's Paradox, though guilty of Russell's Paradox.

I. POSITIVISM AS A CLOSED SYSTEM

What is positivism? In recent decades it has been difficult to say. But perhaps the core belief (erroneous, in my view) is that law is not necessarily connected with morality.[7] What is morality? Scandalously, positivim fails to provide an adequate definition beyond the claim that law is not necessarily connected with it. For the purposes of this Article, I follow Kant in defining morality as normative statements about human

7. W.J. WALUCHOW, INCLUSIVE LEGAL POSITIVISM 166 (1994) ("the distinguishing feature of inclusive positivism is the claim that the moral considerations can, but need not, figure properly in determinations of laws, i.e., attempts to determine the existence or content of valid laws").

behavior that are universally valid and that are derived from universal reason. So conceived, all discourses have a moral dimension, including mathematical discourse. Set theory, after all, recommends that no reference be made to totality (lest contradiction arise). This is purely a normative (and categorical) imperative. Set theory does not deny the existence of totality or of contradiction. It condemns thought that falls prey to it.

Even the core belief that law is not necessaily connected to morality has been denied by some self-described positivists.[8] But positivism must stand for *something*. How can I (or why should I) oppose a philosophy that stands for nothing? So I will operate on the assumption, perhaps incorrect, that positivism still asserts that law and morality are not necessarily connected. But this in turn requires an intensive jurisprudence grounded in rules of recognition. Intension is needed to assure the possibility of a set of primary rules none of which has an origin in morality. Those who assert the disjunctivity (possible or actual) of law and morality are wedded to intensive jurisprudence and therefore to contradiction. The demonstration of the contradiction proves that law *cannot* be separated from morality.

Are there any empirical positivists who think positive law is a closed system? To show that I am not just arguing against nobody, I review a few notorious positivisms to discern whether they can fairly be said to advocate the possibility of a purely intensive jurisprudence. The result demonstrates that positivism does exists empirically.

A. Kelsen

Probably the granddaddy of modern legal positivism is Hans Kelsen, whose *Pure Theory of Law* purports to yield a scientific description of it.[9] But Kelsen by no means closed the system. Indeed, his most famous idea—the basic norm, or *Grundnorm*—stands for an inability to close the system. In effect, Kelsen agreed with Kant, who wrote:

> One can therefore conceive of external lawgiving which would contain only positive laws; but then a natural law would still have to

8. Jules L. Coleman, *Methodology*, *in* THE OXFORD HANDBOOK OF JURISPRUDENCE AND PHILOSOPHY OF LAW 328 (Jules L. Coleman & Scott Shapiro eds., 2002); *see also* JULES COLEMAN, THE PRACTICE OF PRINCIPLE: IN DEFENSE OF A PRAGMATIST APPROACH TO LEGAL THEORY 161 (2001); *but see* Jules Coleman, *Rules and Social Facts*, 14 HARV. J.L. & PUB. POL'Y 703, 711, 716 (1991) ("Though no proponent or critic of legal positivism denies that positivism is committed to the separability thesis, there is far less agreement about what it means . . . ").

9. HANS KELSEN, PURE THEORY OF LAW 218 (Max Knight trans., 2d ed. 1967).

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> precede it, which would establish the authority of the lawgiver (i.e., his authorization to bind others by his mere *choice*).[10]

Kelsen's basic norm is in fact no norm at all, but an absence where a norm should be. Kelsen writes:

> According to a positivistic theory of law the validity of positive law rests on a basic norm, which is not a positive but a presupposed norm, hence not a norm of the positive law whose validity is founded on the basic norm. . . .[11]

The best way to think of the basic norm is to imagine back when, as a child, you continuously asked "why?" when ordered to do something painful. Kelsen himself gives the example:

> For example: A father orders his child to go to school. The child answers: Why? The reply may be: Because the father so ordered and the child ought to obey the father. If the child continues to ask: Why ought I to obey the father, the answer may be: Because God has commanded "Obey Your Parents" and one ought to obey the commands of God. If the child now asks why one ought to obey the commands of God, that is, if the child questions the validity of this norm, then the answer is that this question cannot be asked that the norm cannot be questioned—the reason for the validity of the norm must not be sought: the norm has to be presupposed.[12]

In Kelsen's jurisprudence, the inability to close the system is prior to the system itself. This undoubtedly explains why Kelsen is so profoundly rejected by Anglo-American jurisprudence, which seeks a final and closed *theory* of law—in which law can be reduced to the rules of its recognition.[13]

B. Hart

Sharply critical of Kelsen, H.L.A. Hart attempted to avoid the infinite regress closed by the *Grundnorm* by dogmatically claiming that law is determined by rules of recognition that simply *are*.[14]

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10. IMMANUEL KANT, THE METAPHYSICS OF MORALS 6:225 (Mary Gregor trans., 1991).
11. KELSEN, *supra* note 9, at 219.
12. *Id*. at 196–97.
13. Michael Steven Green, *Hans Kelsen and the Logic of Legal Systems*, 54 ALA. L. REV. 365 (2003) ("Appreciation of Kelsen's work has been further hampered by his notoriously obscure Kantian methodology").
14. SCOTT SHAPIRO, LEGALITY 84, 90 (2011) ("The rule of recognition validates, but is not itself

According to Hart, law is a combination of primary rules that command, authorize or prohibit behavior, coupled with secondary rules of recognition that permit officials to distinguish primary rules[15] from other norms. Rules of recognition are therefore similar to mathematic functions; they select a range of members (primary rules and also the secondary rules other than the rule of recognition) from a larger domain—the set of all norms.[16]

But did Hart imagine that no law exists unless it comports with a prior rule of recognition that grounds it? Perhaps not. Rather, Hart spoke of the "uncertainty . . . of the rule of recognition and so of the ultimate criteria used by courts in identifying valid rules of law."[17]

To understand the nature of this uncertainty, it is necessary to grasp Hart's notion of the legal penumbra. Every primary rule has a core in which application is certain. But this core gives out; beyond the core is the penumbra where application is uncertain. With regard to penumbrae surrounding the primary rules, judges must use discretion. Here judges do not follow law; they make it.[18] I will call this the first-order penumbra.

It turns out that rules of recognition also have penumbrae. These I will call second-order penumbrae. Imagine that the rule of recognition is "whatever Parliament enacts is law." Hart says that doubts can arise about what it means to "enact."[19] At times, second-order doubt resembles the doubts located in the first-order penumbra:

> Sometimes the resolution of [second-order doubt] requires only the interpretation of another rule of law which conferred the legislative power, and the validity of this may not be in doubt. This will be the case, for example, where the validity of an enactment made by a subordinate authority is in question, because doubts arise

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validated . . . the rule of recognition is never valid—it exists . . .").

15. The rule of recognition selects not only to primary rules but also secondary rules of adjudication, execution and change. For ease of elocution I will lump together law recognized by a rule of recognition as "primary rules."

16. FREDERICK SCHAUER, PLAYING BY THE RULES: A PHILOSOPHICAL EXAMINATION OF RULE-BASED DECISION-MAKING IN LAW AND LIFE 199 (1991) ("To the positivist, there can be systems whose norms are identified by reference to some identifier that can distinguish *legal* norms from other norms, such as hose of politics, morality, economics, or etiquette. This identifier, which Hart refers to as the 'rule of recognition' . . . picks out legal norms from the universe of norms, and thus provides a test for legal validity.").

17. H.L.A. HART, THE CONCEPT OF LAW 148 (2d ed. 1994).

18. *Id*. at 251–52 (postscript).

19. *Id*. at 148.

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> as to the meaning of the parent Act of Parliament defining the subordinate authority's legislative powers.[20]

Hart writes that such doubt "raises no fundamental questions."[21]

Hart distinguishes such minor doubt from the doubt concerning the "competence of the supreme legislature itself."[22] In particular, Hart considers whether Parliament could enact a law prohibiting itself from enacting a law. Hart refers to this concept as "legislative suicide."[23] I will borrow Hart's idea of legislative suicide to show that legal positivism succumbs to Russell's Paradox.

Whether a legislature may or may not commit suicide, Hart believes, is a legal question that must be settled by the rule of recognition.[24] Ultimately, whether Parliament can commit suicide is "an empirical question."[25] It is not a moral principle finding its origin in universal reason.

Yet, where the rule of recognition says nothing about suicide, "[q]uestions can be raised about it to which at present there is no answer which is clearly right or wrong. These can be settled only by a choice, made by someone to whose choices . . . authority is eventually accorded."[26] Such a conclusion is reminiscent of Kelsen's basic norm (which Hart supposedly opposes but covertly relies upon).

In the second-order penumbra, "at any moment a question may arise as to which there is no answer—only answers."[27] At such moments:

> The truth may be that, when courts settle previously unenvisaged questions concerning the most fundamental constitutional rules, they *get* their authority to decide them accepted after the questions have arisen and the decision has been given. Here all that succeeds is success. [Sometimes] a very surprising piece of judicial law-making concerning the very sources of law may be calmly "swallowed." Where this is so, it will often in *retrospect* be said, and may genuinely appear, that there always was an "inherent" power in the courts to do what they have done. Yet this may be a

20. *Id*. at 148.
21. *Id*.
22. *Id*.
23. *Id*. at 151.
24. *Id.* at 149.
25. *Id*. at 150.
26. *Id*.
27. *Id*.

> pious fiction, if the only evidence for it is the success of what has been done.[28]

These comments suggest that there is no complete set of rules of recognition that can, in turn, close the set of primary rules.[29] So, on my stipulated definition of positivism, the prophet of Anglo-American legal positivism does not qualify as a positivist at all.

If Hart had left off with the first-order penumbra, he would be guilty of closing the system, with the proviso that some "cases" fall outside of law.[30] Such a vision, as we shall see, falls prey to Russell's Paradox. But what I have described seems not to be Hart's view, because of his idea of the second-order penumbra.

The second-order penumbra must differ from the first-order penumbra. In the first-order penumbra, the judges perform a legislative function. They write a rule and then apply it retroactively to the case before them. A judge, however, does not legislate primary rules in the second-order penumbra, precisely because rule-devising is the very definition of the first-order penumbra. Rather, judges recognize that norms that seem to conform to the existing rule of recognition are in fact not primary rules after all.

Let us consider more carefully what a second-order penumbra must mean. Imagine the set of all normative statements, some of which are the primary rules of law and some of which are non-legal norms. Here are two possibilities:

> (1) A norm complies with a rule of recognition and is a primary rule.
>
> (2) A norm does not comply and is not a primary rule.

These two principles would constitute a genuine positivism. If Hart stopped here, he would be guilty of Russell's Paradox. But Hart can be read as also admitting the existence of two more alternatives:

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28. *Id*. at 153.

29. *Id*. at 148 ("In the overwhelming majority of cases the formula 'Whatever the Queen in Parliament enacts is law' is an adequate expression of the rule as to the legal competence of Parliament, and is accepted as an ultimate criterion of the identification of law, however open the rules thus identified may be at their periphery. But doubts can arise as to its meaning or scope; we can ask what is meant by 'enacted by Parliament' . . . .").

30. Set theorists would call Hart's core "finite support" for a relation between "cases" and outcomes. RAYMOND M. SMULLYAN & MELVIN FITTING, SET THEORY AND THE CONTINUUM PROBLEM 17, 280 (1996). Outside the finite support there is unruliness where the one-to-one correspondence fails.

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(3) A norm may comply with a rule of recognition but, because of its impermissible content, is not a primary rule. The primary rule is invalidated in the penumbra of the rule of recognition.

(4) A norm does not comply with the rule of recognition but is nevertheless a primary rule. It is therefore recognized according to no rule at all (i.e., unruly recognition).

These four propositions can be arranged on a matrix. Each can be characterized as either a positivist or natural law proposition:

A norm:	Is a Primary Rule	Is not a Primary Rule
Complies with a Rule of Recognition	(1) Positivism	(3) Natural Law
Does not comply with a Rule of Recognition	(4) Natural Law	(2) Positivism

When positivism affirms (1) and (2), it asserts a closed system of conventional law and is subject to Russell's Paradox. A closed system disaffirms (3) and (4), because these propositions suggest that some laws are necessarily grounded in morality.

Hart explicitly affirms at least (3). Hart writes, "The words of a statute and what it requires in a particular case may be perfectly plain; yet there may be doubts as to whether the legislature has power to legislate in this way."[31] He does not mention (4).[32] As an example of (3), Hart imagines that the rule of recognition is "only that which Parliament enacts is a primary rule." But some of the norms passed by Parliament are nevertheless not primary rules. In particular, suppose Parliament passes this primary rule: "Parliament is forbidden to enact any primary rule." *This* norm (parliamentary suicide) might be in the penumbra of the rule of recognition.[33] If the suicide provision is an invalid enactment of Parliament, it is so because a court has so found it according to no rule at all. What otherwise seems a primary rule is recognized as not being one.[34]

31. *Id*. at 148.

32. Oddly, Joseph Raz directly affirms (4). "So long as the rule is incomplete some such problems will remain unanswered, but when the courts are faced with such unsolved problems and accept a certain solution they modify the rule of recognition. This should surprise no one. The rule of recognition, being a customary rule, is constantly open to change." JOSEPH RAZ, THE AUTHORITY OF LAW 94 (1979).

33. HART, *supra* note 17, at 150 ("Questions can be raised about it to which at present there is no answer which is clearly right or wrong").

34. Marmor maintains that only what Parliament says is law. The court's refusal to recognize entrenchment is a moral rule. Andrei Marmor, *Exclusive Legal Positivism, in* THE OXFORD

In effect, a court in the above example must flesh out the rule of recognition that was previously thought to exist. Now the rule of recognition has become "whatever Parliament enacts, provided that it does not enact a self-referential rule about its own enactments." There are two difficulties for positivism so refined. First, it suggests there is no complete rule of recognition. Rather, judges do what judges do, and the rule of recognition is just an *ex posteriori* attempt to describe the practice. Such a rule in no way constrains practice. Although this adheres to the truth of how rules actually function,[35] an open-ended rule of recognition that is subordinate to practice is no positivism. Such a theory cannot sustain the disjunction of law and morality. The rule against legislative suicide, for instance, must have a supraconventional origin that is not somehow the subjective whim of the judge (for otherwise we would be in the first order penumbra).

Hart locates the anti-suicide provision that constrains Parliament inside England's existing rule of recognition.[36] In fact, this location is arbitrary. It could equally be viewed as a primary rule directed at Parliament itself.[37] If so viewed, then a primary rule exists that never complied with the rule of recognition. In short, we could have an instance of (4) in the above matrix, not (3). Accordingly, I claim that (3) always implies (4), if we choose to locate the restriction on Parliament in the set of primary rules.

If (3) and (4) are part of the system, then law is generated by a moral intuition that is not part of the rule of recognition. Under such a regime, morality engenders at least one member in the set of primary rules. Which means that Hart is no positivist, as I have defined the term.[38] For instance, it would be incorrect to call Hart a *soft* positivist because this posititon asserts the possibility of hard positivism. This is so because only in a hard positivism can the *possibility* of law's separateness from morality be

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HANDBOOK OF JURISPRUDENCE AND PHILOSOPHY OF LAW 118 (Jules L. Coleman & Scott Shapiro eds., 2002).

35. I develop this theory of rules in David Gray Carlson, *Hart* avec *Kant: On the Inseparability of Law and Morality*, 1 WASH. U. JUR. REV. 21 (2009); David Gray Carlson, *The Traumatic Dimension in Law*, 24 CARDOZO L. REV. 2287 (2003). *See also* STANLEY FISH, DOING WHAT COMES NATURALLY 510–11 (1989).

36. *Id*. at 149–50.

37. SHAPIRO, *supra* note 14, at 80 (power-conferring rules and duty-imposing rules equated as primary rules under the secondary rule of recognition); *see also id.* at 98 ("primary legal rules . . . confer powers . . . ").

38. For a rather different view, see WALUCHOW, *supra* note 7, at 4, 180 (". . . all defenders of positivism reject the notion of criteria of . . . legal validity inherent in something outside of particular legal systems and their accepted tests for validity"). Hart's second-order penumbra precisely allows in something outside of particular legal systems, if "legal systems" is the set of secondary rules of recognition.

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manifested as an actuality. Yet, where the rule of recognition has a penumbra, no hard positivist regime can be derived.

At one point, however, Hart seems to hint that the legal system might be closed. Hart says that, in English society, (3) has been excluded by incorporating within the rule of recognition the premise that Parliament may not commit institutional suicide:

> That parliament is sovereign in this sense may now be regarded as established, and the principle that no earlier Parliament can preclude its "successors" from repealing its legislation constitutes part of the ultimate rule of recognition used by the courts in identifying valid rules of law.[39]

Hart goes on to say that "no necessity of logic, still less of nature, dictates that there should be such a Parliament; it is only one arrangement among others, equally conceivable, which has come to be accepted . . . ."[40]

If this constitutes a statement that closure of the system is logically possible, then perhaps Hart is a positivist after all and therefore guilty of Russell's Paradox. The question therefore devolves into whether Hart thinks penumbrae are mere empirical phenomena or are logically required. If penumbrae can be eliminated at both the primary and secondary levels, then Hart must be counted as a positivist and therefore subject to Russell's Paradox. On this fundamental matter Hart is simply equivocal.

C. Raz

Hart's jurisprudence engendered the fecund critique of Ronald Dworkin, who accused positivism of having a closed system that entirely excluded morality. This was what Dworkin meant by his mysteriously presented "semantic sting."[41] As a result of Dworkin's various critiques, positivism split into hard and soft positivism. The hard positivists claim

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39. HART, *supra* note 17, at 149.

40. *Id.*

41. For my interpretation of what Dworkin intends by "the semantic sting," see David Gray Carlson, *Dworkin in the Desert of the Real*, 60 U. MIAMI L. REV. 505 (2006). Quine anticipated the semantic sting: "In so far as our basic controversy over ontology can be translated upward into a semantical controversy about words and what to do with them, the collapse of the controversy into question-begging may be delayed." WILLARD VAN ORMAN QUINE, FROM A LOGICAL POINT OF VIEW 14 (1953).

that law must exclude morality as a source of primary rules. Its chief progenitor is supposed to be Joseph Raz, who wrote:

> A social condition is necessary for identifying the existence and content of the law: A rule is a legal rule only if it meets a social condition . . . . A social condition is sufficient for identifying the existence and content of the law: A rule is a legal rule if it meets the social condition.[42]

As social conditions are necessary and sufficient, law must be a closed system.[43] Here, "social condition" means a rule that originates in human legislation (which is extended to include custom, on the unproven dogma that customs always have human origins). It is meant to exclude morality, whose origin is in reason and/or God. But origin in godless unreason and human pathology is not enough. Hard positivism also insists that the rule of recognition be *accepted*. That is, it must be conventional.[44] Society must believe that the rule of recognition governs generally.

In the passages that establish the hard positivist manifesto, Raz explains why the "sources thesis" is superior to Dworkinian jurisprudence and to soft positivism. According to Raz, the function of law is to give guidance.[45] When law gives guidance, it has *authority*.

To understand Raz's concept of authority, we must first describe Raz's underlying psychological theory. According to this theory, human acts are rationally caused (or at least potentially so). Prior to acting, a person assembles all the reasons that are relative to him. These are "weighed,"[46] and what emerges is a rational decision to which the person is bound. The actor, however, is imperfect in practical reasoning and knows it. Someone else may know better how to enumerate and weigh the reasons than the actor himself. Authority is the acknowledgement of someone else's superior practical reason.

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42. RAZ, AUTHORITY, *supra* note 32, at 40; *see also id*. at 97 (rules of recognition are "the only ultimate laws that necessarily exist in every legal system").

43. *But see* Joseph Raz, *Two Views of the Nature of the Theory of Law: A Partial Comparison*, *in* HART'S POSTSCRIPT: ESSAYS ON THE POSTSCRIPT TO THE *CONCEPT OF LAW* 1, 9, 10 (Jules Coleman ed. 2001) (stating that "necessary and sufficient" conditions can produce vagueness, that essential characteristics might be neither necessary nor sufficient, that concepts can have more than one set of necessary and sufficient conditions, and that explanations are "almost always" incomplete).

44. JULES COLEMAN, MARKETS, MORALS AND THE LAW 4 (1988) ("A social rule is one whose authority is a matter of convention").

45. For a dissenting view, see Marmor, *supra* note 34, at 106–07.

46. In Raz's universe, authority is not absolute but varies in intensity. JOSEPH RAZ, THE MORALITY OF FREEDOM 100 (1986).

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Authority is defined as having three elements:

> (1) *The dependence thesis:* The norm giver has considered all the reasons relevant to the actor; the norm is "dependent" on these underlying reasons.
>
> (2) *The normal justification thesis*: The actor thinks that an authority knows better than the actor what is good for the actor.
>
> (3) *The pre-emption thesis*: The directive replaces some (but not all) of the reasons the actor has assembled.[47]

In Raz's vision, the actor thinks that law embodies wisdom (the normal justification thesis). Law is not followed just because it is law (as in Hart's internal point of view) but because it plays a part in private rational calculation.

The reason law must have a source in human legislation, Raz thinks, is that, whenever moral intuition produces the primary rules of law, the primary rule emerges only after the actor acts. This after-the-fact revelation of law can provide no guidance. Law in this superegoic form can play no part in the practical reason of the actor. From this it supposedly follows that law must have a human legislative source. Positive law can be researched. Law stemming from the spontaneous moral intuition of the judge cannot be researched. In Raz's universe, we cannot, apparently, predict what judges will do.[48]

Important at present is that Raz's argument seems to be conditional in nature: *if* law is to guide, *then* hard positivism is required. Whether law can be grounded in that which is useful for practical reason is beyond the scope of the present endeavor, which is to show that any closed conventional legal system is contradictory. I don't read Raz as insisting that law *necessarily* guides.[49] And rightly so. Often law is too vague to guide.

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47. This is my paraphrase of JOSEPH RAZ, ETHICS IN THE PUBLIC DOMAIN 198 (1994).

48. *See* WALUCHOW, *supra* note 7, at 121–22 (criticizing the assumption that legal results are clear and moral intuitions are unpredictable).

49. *But see id*. at 120 ("As for Raz, it is clear that his ascriptions of purpose are descriptive in nature and based not on moral-evaluative ascriptions of worthy purposes but meta-theoretical judgments of importance"). Fred Schauer is closer to the point. According to Schauer, all Raz means is that some legal systems are "accurately *described* as positivistic." SCHAUER, *supra* note 16, at 198. That is, some systems guide, but this is no necessary feature of law.

In any case, Raz writes many things that suggest that law is an *open* system. For example:

> Attempting to formulate criteria of validity based on complex court practices that are in a constant state of change and that are necessarily vague and almost certainly incomplete, involves not only legal perceptiveness and theoretical skill, it demands sound judgment and reasonable value-decisions as well. Hart's theory leads one to the point where the boundaries between analytic and critical jurisprudence . . . begin to blur.[50]

Such a passage coheres with a view that says legal practice can never be reduced to rules. Rather, practice precedes the rules (i.e., extensive theory). Raz also writes:

> If the theory of the rule of recognition is substantially correct, as I think it is, it forms part of the answer to the problem of identity. Although it sets necessary conditions for membership in a legal system, it does not provide all the sufficient conditions.[51]

If the rule of recognition is not sufficient, the set of primary rules is not closed. And, contra to Raz, the theory of the rule of recognition is quite *incorrect*. A theory is either correct or not. Quantifying a theory ("substantially correct") is to announce that the theory is not falsifiable and therefore not a theory.

Raz also does not seem to protest the connection of morality with law, so long as it is conceded that moral principles are:

> derivative characteristics of law. If all legal systems necessarily possess certain moral characteristics they possess them as a result of the fact that they have other properties which are necessary for them to fulfil their unique social role.[52]

This seems to say that perhaps all legal systems have primary rules engendered by moral intuition. But moral intuition is not enough. (For example, the moral intuition must perhaps be that of an official.)

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50. RAZ, AUTHORITY, *supra* note 32, at 94. Raz further indicates that primary rules can be valid under rules *other* than the rule of recognition. He gives as an example, legislated rules, which are valid by virtue of "the laws conferring legislative powers . . . ." *Id*. at 95. So for Raz, "all norms legislated by Parliament is law" is *not* a rule of recognition. This makes little sense, where Parliament confers the rule-making power.

51. *Id.* at 97.

52. *Id*. at 104.

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So if Raz is to be considered a hard positivist, it can only be in the sense that it is "hard" to see why this conclusion is justified. Although Raz at one point proclaims the sources thesis to be necessary and sufficient to close the set of primary rules, he also says many things that contradict to this point. Ergo, it is impossible to claim with confidence that Raz is a positivist, as I have defined it. The most that can be said is that he is occasionally so.

D. Coleman

Hart and Raz may forfeit the name "positivist" because they only occasionally reduce law to the rules by which individual laws are recognized. Jules Coleman, also occasionally, proclaims that law can be reduced to the rules of its recognition. Ironically, Coleman is the leader of the soft positivists while remaining one of the few positivists who has unambiguously stated that hard positivism is possible.

Soft positivism holds that law can (but need not) exclude morality. Yet, Coleman offers the clearest statement of hard positivism that I have been able to find. In pursuit of the transcendental essence of law,[53] Coleman puts forth two different principles: negative positivism and incorporationism.

Incorporationism is beyond the scope of this Article. It stands for the possibility that law can be *completely* determined by moral intuition. This argument of a closed system of moral intuition is itself contradictory, but on grounds different from Russell's Paradox. This demonstration must be deferred to another day.

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53. For Coleman, "the fundamental question of analytic jurisprudence [is] what, if anything, is essential to our concept of law . . . ?" COLEMAN, PRINCIPLE, *supra* note 8, at 216; *see also* RAZ, AUTHORITY *supra* note 32, at 104 (legal philosophy must be concerned with the "necessary and universal" features of legal systems). We may pause for a moment to consider how far positivism has come from the days of A.J. Ayer, who wrote the principal manifesto of the logical positivists. The logical positivists of the Vienna circle named themselves as such because they believed in only two forms of truth—logical validity (i.e., tautology) and empirical observation. REBECCA GOLDSTEIN, INCOMPLETENESS: THE PROOF AND PARADOX OF KURT GÖDEL 84–90 (2005). Any reference to mental states or morality was what A.J. Ayer called *nonsense*, because these are facts that cannot be observed. According to Ayer:

> [W]e shall maintain that no statement which refers to a "reality" transcending the limits of all possible sense experience can possibly have literal significance; from which it must follow that the labours of those who have striven to describe such reality all been devoted to the production of nonsense.

A.J. AYER, LANGUAGE, TRUTH AND LOGIC 34 (1936). Positivism's attention to essence is no claim from *ex posteriori* knowledge (i.e., based on experience). It is rather a claim to what Kant would have called synthetic *a priori* knowledge. Coleman and Raz are not in the business of sociological description but in the transcendental deduction trade.

Our current concern is solely with Coleman's negative positivism. Why *negative* positivism? "Instead of articulating some truth about all law everywhere, negative positivism simply denies that morality is necessarily a condition of legality for all possible legal systems."[54]

Negative positivism is organized by a limit case in which all law has "pedigree"—that is, traceable to human legislation. "*[T]here exists at least one conceivable rule of recognition (and therefore one possible legal system) that does not specify truth as a moral principle among the truth conditions for any proposition of law.*"[55] This proposition, Coleman suggests, is "conceptually unassailable and descriptively accurate. There is no logical . . . contradiction in asserting that there exists a possible world in which there is law and in which what makes something law is not a matter of its morality."[56]

Coleman often insists (dogmatically, by the way) that rules of recognition must necessarily be there. "We need a theory of law that merges the idea of a rule of recognition with the view that moral principles can someimes be binding on officials," Coleman writes.[57] "Incorporationism responds to legal positivism's ambitions as a general, descriptive jurisprudence. It asserts that wherever there is law, there is a social rule (the rule of recognition) that sets out criteria of legal validity."[58] The rules of recognition are "the signature of a legal system."[59]

Here we have grist for the grindstone of Russell's Paradox, which shows that negative positivism is *completely* assailable, *contradictory* to the core, and descriptively *inaccurate*. To be sure, Coleman's claim is only that such a regime is possible, not that it exists or has existed or ever will exist.[60] Possibility encompasses non-existence as well as existence. It

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54. Coleman, *Social Facts*, *supra* note 8, at 716.

55. COLEMAN, MARKETS, *supra* note 44, at 5 (emphasis added). Elsewhere, Coleman writes:

> Though no proponent or critic of legal positivism denies that positivism is committed to the separability thesis, there is far less agreement about what it means and its implications for positivism . . . I understand this to mean that positivism is committed to the view that the morality of a norm is not necessarily a condition of its legality.

Coleman, *Social Facts*, *supra* note 8, at 716.

56. *Id.*

57. Jules Coleman, *Incorporationism, Conventionality, and the Practical Difference Thesis*, *in* HART'S POSTSCRIPT, *supra* note 43, at 99, 129 n.45.

58. *Id.* at 145.

59. COLEMAN, PRINCIPLE, *supra* note 8, at 76.

60. COLEMAN, MARKETS, *supra* note 44, at 13–16; Jules Coleman, *Reason and Authority*, in THE AUTONOMY OF LAW: ESSAYS ON LEGAL POSITIVISM, 287, 291 (Robert P. George ed., 1996) ("The separability thesis is not the claim that law and morality are necessarily separated; rather, it is the claim that they are not necessarily connected"); *see also* E. Phillip Soper, *Legal Theory and the Obligation of a Judge: The Hart/Dworkin Dispute*, 75 MICH. L. REV. 473, 512 (1977).

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excludes only impossibility and contradiction.[61] What Russell's Paradox will show is that Coleman's negative positivism is indeed impossible.

Coleman, however, has lately repented of his positivism, according to a recent co-authored piece:

> Too many commentators have understood the project of jurisprudence to be an effort to identify a set of necessary and sufficient conditions for being law or necessary and sufficient conditions for being the concept LAW. Many of the features of legal practice that are most illuminating for what is to be law, or to be laws, are neither necessary nor sufficient conditions of legality. Hart understood this as well as anyone and displayed his sensitivity to it in his jurisprudence. That he is associated with the efforts to articulate a set of necessary and sufficient conditions that define the concept LAW or provide a definition of "law" remains not only a puzzle but something of an embarrassment to the philosophy of law.[62]

As with Raz, it is no longer certain that Coleman can be listed among the positivists, as he does not always maintain that law has necessary and sufficient conditions of existence.

E. Rawls

One does not ordinarily think of John Rawls as a jurisprudential theorist, but in his famous essay, *Two Concepts of Rules*,[63] he adopts an intensive jurisprudence by implication. In this essay, Rawls proposes that there are two concepts of rules. The first is the so-called summary concept: rules summarize past practices. This is consistent with what I have called extensive jurisprudence. In this view, "cases are logically prior to rules."[64] The other view is the practice concept: rules of practice are "logically prior

61. Rather wickedly, Matthew Kramer points out that, on Coleman's reading, a legal system that consists of judges who decide cases while under the delusion they were acting out Hamlet in a play is consistent with positivism because I can imagine it. Matthew H. Kramer, *On Morality as a Necessary or Sufficient Condition for Legality*, 48 AM. J. JURIS. 53, 71–76 (2003).

62. Jules L. Coleman & Ori Simchen, *"Law"*, 9 LEGAL THEORY 1, 41 (2007); *cf.* COLEMAN, MARKETS, *supra* note 44, at 3 ("Every theory about the nature or essence of law purports to provide a standard, usually in the form of a statement of necessary and sufficient conditions, for determining which of a community's norms constitute its law").

63. John Rawls, *Two Concepts of Rules*, 64 PHIL. REV. 3 (1955).

64. *Id.* at 22.

to particular cases."[65] This latter model of rules closes the system. It reduces practice to the rules of recognition, and therefore is a legal positivism as I have defined it. These are "rules in the strict sense."[66] The summary view entails "rules of thumb"[67]—not really rules at all but merely a prediction of what an actor will do. Rawls's theory of rules in the strict sense therefore falls prey to Russell's paradox.

F. Marmor

Another positivist who argues that the system *must* be closed (a self-styled hard positivist) is Andrei Marmor. For Marmor all rules of recognition are conventional. And all applications are guaranteed. If there is any controversy, then it follows there is no convention and therefore no law. Marmor's complaint against soft positivism is:

> [I]t assumes that there is a potential gap between the convention, which is a rule, and its application, a gap that can be bridged by moral and political arguments. The main reply to this is, that there is no such gap. The convention is constituted by the practice of its application to particular cases. It is not the case that we first have a rule-formulation, say that convention "R" prescribes so and so, and then we try to make up our minds how to apply R to particular cases (and then, as this story goes, sometimes we know the answer, and sometimes we argue about it). Conventions are what they are, because there is a practice of applying the rule to certain cases; it is the application of the rule which constitutes its very existence. Once it is not clear . . . whether the convention applies to a certain case or not, then there is no conventional solution to that matter, and at least as far as the convention is concerned, this is the end of it.[68]

We have, then, a representative of hard positivism who falls prey to Russell's Paradox. For Marmor, either there is convention (and hence law), or there is morality and politics (and hence no law). The two sets of norms are conceived as disjoint.

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65. *Id.* at 25.
66. *Id.* at 29.
67. *Id.* (internal quotations omitted).
68. Marmor, *supra* note 34, at 112.

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G. Waluchow

Another scholar who seems to close the system is W.J. Waluchow, who writes:

> [T]he positivist's thesis [is] that a legal system contains a *finite* set of special rules distinguishable according to some master text, from all other rules which might be in some way applicable to human behaviour.[69]

And:

> There is no law beyond or above the law as Dworkin and defenders of natural law theory would have it. The law is just that determinate and limited set of rules satisfying a socially constituted rule of recognition.[70]

Waluchow, then, agrees with my definition of positivism. If the system isn't closed, then it isn't positivism. For Waluchow, if the number of primary rules is finite, then either (a) the primary rules are overdetermined by the rules of recognition (which positivism usually denies), or (b) the number of rules of recognition are themselves finite. If (b) is correct, the system is closed, and Russell's Paradox applies.

H. Schauer

Our final empirical example of a positivist is Fred Schauer:

> Imagine a system, however, in which (a) a master rule of recognition pedigreed a limited number of rules; (b) all decision-makers within the system were instructed to make decisions according to and only according to those pedigreed rules; and (c) a default (or closure) rule specified the result to be reached in all cases not covered by one of the pedigreed rules. In such a system, a positivistic explanation appears to be sound, and we can thus see that positivism as a descriptive thesis is flawed not conceptually . . . but only empirically. Few legal systems resemble this stylized model, but *there is no logical reason why they could not*.[71]

69. WALUCHOW, *supra* note 4, at 34–35.
70. *Id.* at 235.
71. SCHAUER, *supra* note 16, at 202 (emphasis added).

In fact, Russell's Paradox provides the logical reason why legal systems cannot resemble (or, shall we say replicate) the closed model.

At least some empirical positivists, then, have closed the system and fallen afoul of Russell's Paradox. These are the jurisprudences that assert the possibility that a rule of recognition might be complete. Several jurisprudences equivocate as to the closure of the system. Accordingly, I am licensed to proceed to the demonstration that Russell's Paradox lurks within at least some positivisms.

II. RUSSELL'S PARADOX

A. The Basic Demonstration

According to positivism, law and morality are not necessarily connected. For this to be true, it must be possible for a rule of recognition to define a set of primary rules such that every single primary rule has its origin in human legislation. It must exclude all *natural* law with an origin independent of arbitrary human will. If this exclusion cannot be accomplished, positivism cannot claim that the set of laws can be entirely disjoint from the set of moral norms.

Russell's Paradox, however, proves that such a disjoint set of purely positive laws is *impossible*. The paradox involves the set of all sets that do not belong to themselves. Here is a helpful verbal account of Russell's Paradox:

> Consider the set S whose elements are all those (and only those) sets that are not members of themselves . . . Question: Does S belong to S? If S belongs to S, then S is not a member of itself . . . On the other hand, if [S does not belong to S], then S belongs to S. In either case, we have a contradiction.[72]

It is this paradox that defeats positivist jurisprudence.

Positivist jurisprudence purports to be a closed set of sets. In fact it must contain at least one subset that does not belong to the larger set.[73] The proposition of positivist jurisprudence can be expressed as follows.

Let w represent a domain of possible litigation outcomes. It includes outcomes that are legally required and outcomes that are merely subjectively desired.

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72. JECH, *supra* note 4, at 4.

73. Analogously, for any class A there is a subclass B of A such that B is not an element of A. SMULLYAN & FITTING, *supra* note 30, at 17.

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Let x represent a set of primary rules. Primary rules select from w those outcomes that are legally required: $w_x \in x$. $w-w_x$ may be conceived as subjectively desired outcomes that are not legally required. x may be conceived as a function on w: $\Phi(w)=x$.

Let y represent the universal set of law, as determined by a rule of recognition: $x \subset y$.[74] y is therefore a set of sets (i.e., each primary rule is a set of legally required outcomes). Set y is to be conceived as the range of a function on x: $y=\Phi(x)$.

Putting these points together, positivism defines law as the set of socially conventional secondary rules $\Phi(x)$ by which officials recognize the set of primary rules.

Russell's Paradox involves self-reference. So we must now imagine that the rule of recognition recognizes *itself* as a law, or $\Phi(x)=x$. Suppose the rule of recognition is "whatever Parliament enacts is law." Under this conjuration, the learned lords enact the following law: "Whatever Parliament passes is law." In such a case, the secondary rule becomes a primary rule.[75] So far, however, we have no contradiction. We have merely a set that belongs to itself. This is usually permissible in set theory. For example, the set of all sets with more than two elements belongs to itself.[76]

Now suppose that Parliament passes *this* law: "Nothing Parliament says or ever will say is law." This is Hart's legislative suicide.[77] Once this

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74. $x \subset y$ is to be read as "x is a subset of y." $\subset$ (inclusion) differs from membership ($\in$) in that $\in$ can refer either to subsets or to "individuals." ALPHA CHIANG, FUNDAMENTAL METHODS OF MATHEMATICAL ECONOMICS 13 (3d ed. 1984). "It is well known that logic and mathematics are expressible in a language comprising only alternative denial . . . , universal quantification, and the predicate $\in$ of class membership." QUINE, *supra* note 2, at 201; *see also id.* at 113 ("we may think of '$\in$' as the one sign for set theory in addition to those of elementary logic").

75. Hart disputes this point. The rule of recognition, he writes, "is not a rule on the same level as the 'laws strictly so called' which it is used to identify. Even if the [rule of recognition] were enacted by statute, this would not reduce it to the level of a statute; for the legal status of such an enactment necessarily would depend on the fact that the rule existed antecedently to and independently of the enactment." HART, *supra* note 17, at 111.

76. Self-membership is sometimes outlawed by the Axiom of Regularity. PATRICK SUPPES, AXIOMATIC SET THEORY 53 (1960); ALAIN BADIOU, BEING AND EVENT 356 (Oliver Feltham trans., 2005) ("ontology does not recognize any event, because it forecloses self-belonging"). Set theory often does without this axiom, useful though it is in defining cardinal numbers without reliance on the Axiom of Choice. JECH, *supra* note 4, at 27, 165–66. In general, any "well founded" set (one with an initial element) outlaws self-membership. SMULLYAN & FITTING, *supra* note 30, at 99–102.

The power set P(x) is based on self-membership. Take a set x of two elements {a,b}. P(x) is the set of all subsets of {a,b}. The size (or cardinality) of P(x) is 2^n, where n is the number of elements in x. CHIANG, *supra* note 74, at 13; SUPPES, *supra* note 76, at 46–47. In a two-element set, there should be four subsets. These are: {$\emptyset$: $\emptyset$ is the null set}, {a}, {b}, {a,b}. Thus, $\{a,b\} \in \{a,b\}$. Otherwise, 2^n does not describe the power set. *Id*. at 47 (theorem 87).

77. *See supra* text accompanying notes 19–23.

primary rule is admitted in the set, law becomes a contradiction. Law can endure *only if this particular primary rule is prohibited*. Yet the prohibition does not conform to the socially recognized rule of recognition ("whatever Parliament says is law").

Sets of sets that do not belong to themselves are contradictory. This is the claim of Russell's Paradox, which revolutionized set theory in 1902. The claim of positivist jurisprudence can be reproduced in the following set theory terms:

$$(\exists y)(\forall x)(x \in y \leftrightarrow \Phi(x)) \qquad (1)$$[78]

In jurisprudential English, (1) states that there exists a legal universe ($\exists y$) such that for every primary rule ($\forall x$) that exists in the universe ($x \in y$), a social rule of recognition (Φ) establishes membership of x in y. The entire phrase (Frege's ill-considered axiom of abstraction)[79] states that there is a legal universe in which every primary rule is grounded in some pre-existing rule of recognition.

In (1), $\Phi(x)$ is the rule of recognition. It selects members in y. For example, suppose $\Phi(x)$ stands for the rule that "whatever Parliament enacts is law." Every x is a norm, and it is a member of y, when Φ selects it. y is therefore a closed system. On (1), y can be viewed as nothing *but* the rule that whatever Parliament enacts is a law. Law so conceived is intensive—reducible to a selecting function (a rule of recognition). When Φ chooses y as the rule, $x=y$ and $y \in y$. This is no contradiction, but it shows that y can be a member of y if Parliament enacts y.[80]

It is also the case that Φ could select the negative of y. Suppose Parliament enacts "nothing Parliament enacts is law," or $\Phi(x) = -(y \in y)$.[81] Substituting this into (1):

$$(\exists y)(\exists x)(x \in y \leftrightarrow -(y \in y)) \qquad (2)$$

78. The symbol $\exists$ means "there exists at least one." The symbol $\forall$ means "for all." Φ represents a function that creates a set by selecting from a larger set according to a rule. For example, Φ could stand for "whatever Parliament enacts is law." Φ would then select from the larger set of all normative sentences those sentences that have been enacted by Parliament into law.

79. SUPPES, *supra* note 76, at 5 ("given any property there exists a set whose members are just those entities having that property"). It is also called the axiom of unrestricted comprehension. TILES, *supra* note 5, at 146. According to this false theorem: If P is a property, there exists a set $Y = \{x: P(x)\}$. JECH, *supra* note 4, at 4.

80. Given (1) $(\exists y)(\forall x)(x \in y \leftrightarrow \Phi(x))$, we can eliminate x by substituting y: $(\exists y)(\forall y)(y \in y \leftrightarrow \Phi(y))$. No contradiction here, if we get rid of the Axiom of Regularity.

81. Such an enactment might also be worded as follows: "Whatever Parliament enacts is not the law."

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In English, this says, "There exists a closed set (y) of rules (x) and in set y there exists the rule -(y∈y)." In other words, x is a rule (because enacted by Parliament) and -(y∈y) has been enacted by Parliament and therefore is an instance of x. So x is *and is not* a primary rule.

In (2), since y=x, we can substitute y for x across the board.

$$(\exists y)(\exists y)[(y \in y) \leftrightarrow -(y \in y)] \qquad (3)$$

In English, (3) means that the proposition of a closed universe logically requires one to believe that everything Parliament passes is law and that *nothing* Parliament passes is law.

$$(y \in y) \leftrightarrow -(y \in y) \qquad (4)$$

By (4), positivist jurisprudence is a contradiction. The difficulty is that y purports to be a set that is not a member of itself. It both is and is not a primary rule.

The implication of this exercise is that positivist law fails as a completed universe. There is at least one *real unconventional rule* organizing the set of primary rules that is not itself a signifier of Φ. In Gödelian terms, there exists a sentence which is not provable in the formal system.[82] That sentence would be "the enactment of -(y∈y) is prohibited." Φ(x) must not be allowed to select -(y∈y).[83]

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82. *See* JECH, *supra* note 4, at 157 ("The First Incompleteness Theorem shows that no consistent (recursive) extension of Peano Arithmetic is complete. In particular, if ZFC is consistent (as we believe), no additional axioms can prove or refute every sentence in the language of set theory."). ZFC refers to the Zermelo-Fränkel axioms, with the controversial Axiom of Choice thrown in. *Id*. at 3.

83. The result reached here parallels that derived in Alfred Tarski, *On Undecidable Statements in Enlarged Systems of Logic and the Concept of Truth*, 4 J. SYMBOLIC LOGIC 105 (1939). Tarski's theorem is that there is no set-theoretical property T(x) such that, if σ is a sentence, T(╔σ╗) holds iff σ holds. JECH, *supra* note 4, at 162. ╔σ╗ represents an encoding of σ within a sequence of all valid sentences in the language of set theory. The proof invokes a sequence of formulas at least one of which commits suicide.

Let T(x) be defined as

$$\forall x(T(x) \rightarrow \{x \text{ is an ordinal number}\}) \;\&\; \sigma \leftrightarrow T(\text{╔}\sigma\text{╗})$$

Imagine an enumeration of all formulas ℘ with one free variable:

$$\wp_0(x),\ \wp_1(x),\ \wp_2(x) \ldots$$

ψ(x) represents a formula programmed to blow up. It stands for

$$\{x: x \text{ is an ordinal number}\} \;\&\; -T(\text{╔}\wp_x(x)\text{╗}).$$

There is a natural number k such that $\psi(x) = \wp_k(x)$. Then

$$\sigma \leftrightarrow \psi(k) \leftrightarrow -T(\text{╔}\wp_k(k)\text{╗}) \leftrightarrow -T(\text{╔}\sigma\text{╗}).$$

Yet T(x) was defined as σ ↔ T(╔σ╗). Hence, σ means T(╔σ╗) and -T(╔σ╗) simultaneously. In this proof ψ(x) is the equivalent of "nothing Parliament says is law."

B. The Axiom Scheme of Separation

In set theory, Russell's Paradox engendered the Zermelo-Fränkel axioms, designed to stave off contradiction by banishing any reference to the set of all sets. Once this is done, the set of all non-self-belonging sets cannot infect the brew. But the price of this banishment is that set theory has no criterial theory of what a set is.[84] In effect, set theory prohibits the theorizing of sets, if "theorizing" means reducing objects to their rules of recognition.

The legislative abolition of the set of all sets that are not members of themselves is accomplished by Fränkel's axiom scheme[85] of separation. The point of this axiom scheme is to prevent closure of the system. So long as the system is not closed, we *can* refer to $-(y \in y)$ without generating the contradiction of (4).[86]

Formally, the axiom scheme of separation is expressed as follows:

$$(\exists y)(\forall x)(x \in y \leftrightarrow (x \in z \leftrightarrow \Phi(x)) \qquad (5)^{87}$$

In jurisprudential English, (5) asserts that there exists a set of laws (y) and there exists a member of that set (x) such that x is a law ($x \in y$). x has been chosen from the set of all possible norms (z). Or, given a formula $\Phi(x)$, for every set y, there exists a set z such that the formula selects for y an element of z.[88] x is selected from some other set z by Φ to be a member of y. But it is important to comprehend that $\Phi(x)$ no longer stands for Parliamentary enactment. Rather, it stands for the rule by which we recognize that a statement (whether enacted by Parliament or not) is normative (i.e., it is a sentence that uses, or could be rewritten to use, "ought"). Imagine that z stands for "all possible normative statements, whether enacted by Parliament or not." Once the axiom scheme of separation is adopted, Russell's Paradox no longer functions. Suppose, as before, Φ recognizes that in z is the principle that nothing Parliament enacts is law $-(y \in y)$. Then $(x \in z \leftrightarrow -(y \in y))$. This is not to say that Parliament has enacted $-(y \in y)$. All Parliament has done is to enact $(x \in y)$.

84. *See* MAXWELL ROSENLICHT, INTRODUCTION TO ANALYSIS 2 (1968) ("We do not attempt to define the word *set*").

85. The word "scheme" is used because the axiom scheme is not a single axiom but is in fact a family of axioms (Φ), any one of which could be chosen according to the form of the scheme. MICHAEL POTTER, SET THEORY AND ITS PHILOSOPHY 13–14 (2004).

86. *See* QUINE, *supra* note 2, at 202 (stating that $-(x \in x)$ "can be ruled out by one or another stipulation; but it is significant that such stipulations are *ad hoc*, unsupported by intuition").

87. SUPPES, *supra* note 76, at 7.

88. JECH, *supra* note 4, at 177.

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It is only the case that $-(y \in y)$ has been recognized (Φ) as a *possible* norm that Parliament *might* enact. In short, we have "Parliament has acted" *and* "what Parliament enacts ought not to be a law." These propositions are quite consistent with each other.[89]

Now if $x=y$, and if we eliminate all x by substituting y, we obtain

$$y \in y \leftrightarrow (y \in z \leftrightarrow -(y \in y)) \qquad (6)$$

In jurisprudential terms, this states that "whatever Parliament enacts is law" is in fact enacted by Parliament ($y \in y$). It further states that if $y \in y$, then, by implication, y is a norm ($y \in z$) and one of the norms in z happens to be "nothing Parliament enacts is law." There is no contradiction here. Parliament might enact y. In addition, it could be true that Parliamentary suicide is a norm that Parliament has not necessarily enacted.

The trick is that in (5), we are given a set z dogmatically ("there is a closed set of norms, whether enacted by Parliament or not"). *Then* we assert the subset y ("whatever norm Parliament enacts is law") and make claims about its membership ($y \in y$).[90] In comparison, (1) asserted y unconditionally and unconventionally. In (1), $-(y \in y)$ could be made into a social rule by Φ. Therefore, y was both a social rule and not a social rule.

It is my contention that Russell's Paradox defeats legal positivism. Positivism simultaneously asserts $(x \in y)$ & $-(x \in y)$.[91] Therefore, the existence of the legal system depends on the legislature not enacting $-(x \in y)$. So an unlegislated primary rule exists: $-(x \in y)$ is not permitted.[92] Yet Parliament has never enacted it. Therefore, the prohibition of $-(x \in y)$ exceeds the stated rule of recognition ("only what Parliament enacts is law"). At least one primary rule with no pedigree in the positivist rule of recognition is required to sustain the system requiring that all primary rules have pedigrees. Inevitably, something *real and universal*—

89. Consistency is a defined term in set theory. Once we assume that a theory (say, ZF) is consistent (something ZF cannot establish for itself), we ask whether it remains consistent if we add another axiom (Φ). Suppose we add Φ to ZF. ZF+Φ is consistent relative to ZF if the following holds: If ZF is consistent, then ZF+Φ is consistent. If, in addition, ZF+($-\Phi$) is consistent, then Φ is *independent* of ZF. JECH, *supra* note 4, at 163 (describing Gödel's Second Incompleteness Theorem).

90. The axiom scheme is therefore sometimes called the subset axiom. TILES, *supra* note 5, at 122.

91. This formulation is correct, though I have heretofore referred to $-(y \in y)$. Since Parliament has enacted $-(y \in y)$, $-(y \in y) = x$, and x is both a member and not a member of y.

92. One prominent positivist, at least, agrees. Leslie Green, *The Duty to Govern* 13 LEGAL THEORY 165, 182 (2007) ("Even if Dicey was right in thinking that the U.K. Parliament could lawfully commit suicide by transferring all its powers to the Manchester Corporation, it can scarcely be denied that this would be regarded as an immoral alienation of its duty to govern") (footnote omitted, citing A.V. DICEY, THE LAW OF THE CONSTITUTION (10th ed. 1959)).

something not conventional—intrudes upon the supposedly conventional system.[93]

C. Responses

The above demonstration shows that positivism is not a tenable enterprise. As many positivists will wish to resist this conclusion, let me anticipate a few responses.

One possible response is that my example shows merely that law depends on will. The legislature must refrain from enacting -(x∈y). So long as it refrains, the legal system exists. Ergo legal positivism is possible. This is a bit like saying that there is nothing wrong with Frege's intensive set theory, so long as Frege does not use the axiom of abstraction to generate the set of all sets that do not self-belong.[94] It is true that subjective inhibition can sustain Fregeian set theory: do not refer to the set of all sets that do not belong to themselves. In effect, that is what the Axiom of Separation establishes. Yet the price of adopting this strategy is that set theory is sustained by will, not grounded in logical principle. That is precisely what Zermelo and Fränkel concluded.[95] The axiom scheme of separation *is* the moral prohibition of the set of all sets that do not belong to themselves. It is moral because *reason itself* (not mere will) requires the prohibition in order to sustain the possibility of a noncontradictory set theory.[96] Similarly, in jurisprudence, Parliament cannot be permitted to

93. Jules Coleman writes that law would not be separate from morality:

> [I]f it were not merely a contingent social fact about all legal systems that their rules of recognition made morality a condition of legality, if, for example, it could not be otherwise, that the concept of law required that law be a matter of morality. Then, the relationship between legality and morality would be intensional. Only then would there be no separation between law and morality of the sort envisioned by the separability thesis.

Coleman, *Social Facts*, *supra* note 8, at 723. Coleman has it exactly backwards. The very *intensionality* of law gives rise to the paradox. Jurisprudence is inevitably extensional—*not* reducible to a rule of recognition. For this very reason and as a fundamental matter of logic, law cannot close itself to morality.

94. This is the strategy that Bentrand Russell followed in his now-forgotten theory of ramified types. TILES, *supra* note 5, at 170–71.

95. QUINE, *supra* note 41, at 14 ("As Fraenkel has put it, logicism holds that classes are discovered while intuitionism holds that they are invented . . . ").

96. An anonymous reviewer of a journal found cause to reject this essay because the remark in the text supposedly demonstrates that I have an idiosyncratic view of morality: how could mathematics be subject to moral principle? In fact, the great mathematician David Hilbert explicitly viewed the axioms of set theory to be moral principles. *See* PAUL J. COHEN, SET THEORY AND THE CONTINUUM HYPOTHESIS 2–3 (1966). Georg Simmel is among those who saw that logic is in the end a moral enterprise. GEORG SIMMEL, THE PHILOSOPHY OF MONEY 438 (Tom Bottomore & David Frisby eds., 2d ed. 1978) (1907) ("one can elude a superior logic only by a stubborn 'I don't want to,' which implies a confession of inferiority").

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commit suicide. Only this rule grounded in universal reason sustains the set of primary rules over time.

A related retort would continue with the idea that *arbitrary will*, not morality, sustains the legal system. If law concededly depends on the will not to enact -(x∈y), perhaps such a will is not a moral will. If so, then law perhaps is logically independent of morality. This strategy will not work. Suppose the will of the legislature is corrupt. Its motive is pathological: it refrains from enacting -(x∈y) only because it selfishly wants to perpetuate parliamentary salaries. Therefore the legal system does not depend on morality. Rather, it depends on a will that might be immoral. Yet it remains true that law's dependence on will is itself a universal, normative truth that describes what the legislature must not do if it is to perpetuate the legal regime. It is a supra-conventional "ought" that *is* the very condition for the possibility of law. Where the will to survive is corrupt, then will is governed by a mere hypothetical imperative. Any such legislative heteronomy is contingent. Being contingent, it cannot last. Given enough time, what *may* happen *will* happen. When the legislature grows tired of its corruption, law must cease.[97] Only a *universal* exercise of will—i.e., morality–sustains a legal system over time.[98]

A third possible response is that, even if positivism is contradictory, it is nevertheless what people mean by the word "law."[99] When it comes to word usage, there is no reason to outlaw contradiction. This is a good point, but it is not a philosophical point. It is a sociological one, and therefore is not admissible in the docket of legal philosophy.[100] That people can survive in a state of cognitive dissonence does not make their bad philosophies into good ones. Empirically, people may continue to

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The anonymous reviewer demonstrates that positivism has no definition of morality—just an unspoken intuition that it has something to do with, say, sexual conduct or honoring thy father and thy mother. This constitutes "confusing moral ideas with moral sentiments." ANGELICA NUZZO, IDEAL EMBODIMENT: KANT'S THEORY OF SENSIBILITY 80 (2008). Only by such tactics can positivism defend itself.

97. As the Politburo learned twenty years ago.

98. Coleman is doubly wrong in implying that "positivism is not wedded to any particular meta-ethical or moral ontology." COLEMAN, PRINCIPLE, *supra* note 8, at 159. Indeed, it wishes to assert the conventionality of law, which *is* a meta-ethical position. "No meta-ethics" is a form of meta-ethics, like it or not. Furthermore, conventionality is a contradiction, so it relies on some meta-ethical position other than conventionalism of which it is unaware. Relevant here is the classic remark of Charles Peirce: "Find a scientific man who proposes to get along without metaphysics . . . and you have found one whose doctrines are thoroughly vitiated by the crude and uncriticized metaphysics with which they are packed." 1 CHARLES SANDERS PEIRCE, COLLECTED PAPERS § 129 (1960).

99. RAZ, AUTHORITY, *supra* note 32, at 49–50; Frederick Schauer, *The Limited Domain of the Law*, 90 VA. L. REV. 1909, 1945–46 (2004).

100. RAZ, AUTHORITY, *supra* note 32, at 44 (legal philosophy must address what is necessary for "all legal systems").

practice Fregean set theory, but that doesn't make it into a good set theory.[101]

A fourth response is a perennial favorite of all philosophers—we knew it all the time! You're not telling us anything we didn't already know! Did not Hart himself introduce the idea of legislative suicide? Hart held that England's rule of recognition has an anti-suicide provision in it, such that judges will not recognize -(y∈y), even if Parliament enacts it.[102] The claim that "Hart knew it all the time" must be rejected. First, Hart (not a positivist as I have defined it)[103] held that such an anti-suicide provision embedded within the rule of recognition was *not logically required*.[104] He said was that England had a social rule of recognition. This social rule happens to conform to what reason, logic and morality require. But this doesn't mean that conventionality is the source of the rule. Conventionally, people may agree that 2+2=4, but conventionality is not the reason why 2+2=4. In any case, Hart erred by insisting that the anti-suicide provision is a matter of mere convention. In fact, it is reason, not convention, that prohibits -(y∈y).

To summarize, positivism fails because practice cannot be reduced to socially determined rules. But the claim of positivism itself is not a socially determined rule. Positivism is a set that does not belong to itself. Does this prove that law and morality are therefore related? Does law logically required morality? The answer is yes. A symbolic system, whether it be set theory or law, always has a fissure in it—the traumatic intrusion of a real (i.e., non-conventional, unsymbolized) thing. This real thing is objective and universal. It is the law itself. And it is the *moral* law. We are, after all, talking about the set of primary rules, and whether it can be determined completely by a determinate rule of recognition. The application of Russell's Paradox shows that at least one member of the set of primary rules must be a universal, unconventional norm—a sentence not provable within the system.

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101. Set theory has not entirely abandoned the set of all sets. Such a concept is called a "proper class," which is not deemed to be a "set." JECH, *supra* note 4, at 6, 8, 58, 65, 67. Except for the "proper class," all classes are sets. *Id*. at 6.

102. HART, *supra* note 17, at 148–49.

103. *See supra* Part II.B.

104. *Id*. at 145 ("It is, however, important to see that no necessity of logic, still less of nature, dictates that there should be such a Parliament").

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III. RAZ'S PARADOX

Before concluding, I would like to discuss Joseph Raz's reading of Dworkin, according to which positivism is saddled with a paradox. The issue is whether Raz has anticipated my use of Russell's Paradox against the coherence of positivism, in which case I am saying nothing original in this Article. Positivism would then be able to claim, "Oh, we knew *that* all along."

My conclusion is that Raz has not done so. Rather, Raz improperly reduces Dworkin to an invalid *quaternio terminorum*. A *quaternio terminorum* is a logical error in the following form: (A) all stars are in heaven; (B) Megan Fox is a star; therefore (C) Megan Fox is in heaven.[105] The error stems from the fact that the word "star" in (A) surreptitiously changes meanings in the minor premise (B).

In his essay, *No Right Answer?*,[106] Dworkin imagines a positivism in which the law of the excluded middle holds. According to the law of the excluded middle, a thing either *is* or *is not*. Nothing can both exist and not-exist. In legal terms, either a person has a duty not to do x, or the person is permitted to do x. One or the other proposition must be true. This, I maintain, is not positivism. Rather, it is a legal system underwritten by an unpositivized moral principle: whatever is not prohibited is permitted.

In what follows, x is a primary rule and $\Phi(x)$ is its rule of recognition. For the sake of exposition, assume that $\Phi(x)$ is "Parliament has enacted x."

In Raz's rendition of Dworkin's critique, positivism is reduced to the following rule of recognition:

$$x \leftrightarrow \Phi(x) \tag{7}$$

This says that, if you observe x in the statute books, you can infer that Parliament enacted x. And, equivalently ($\leftrightarrow$), if you sit in the gallery and observe Parliament passing x ($\Phi(x)$), you can be sure that x will appear in the statute books. To use Raz's example, if x stands for "kill no butterflies," it must be true that Parliament has enacted a law commanding "kill no butterflies."

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105. *See* IRVING M. COPI & CARL COHEN, INTRODUCTION TO LOGIC 206–08 (11th ed. 2002). According to Kant, "Such a syllogism is a *paralogism*, in so far as one deludes oneself by it; a *sophism*, in so far as one deliberately tries to delude others by it." IMMANUEL KANT, LOGIC 138 (Robert S. Hatman & Wolfgang Schwarz trans., 1974).

106. Ronald Dworkin, *No Right Answer?*, *in* LAW, MORALITY, AND SOCIETY: ESSAYS IN HONOUR OF H.L.A. HART 58 (P.M.S. Hacker & J. Raz eds., 1977).

The "counterposition"[107] of (7) is

$$-x \leftrightarrow -\Phi(x) \tag{8}$$

This says that if you fail to find x in the books, Parliament has not acted.

The illegitimate move occurs in the next step, where we are to imagine that Parliament has affirmatively enacted the proposition that x ("kill no butterflies") is not to appear in the statute book.

$$-x \leftrightarrow \Phi(-x) \tag{9}$$

In this formulation, -x simultaneously means "there is no enactment concerning butterflies" and "it is permitted to kill butterflies." -x therefore changes meanings in midstream. On the one hand, -x is a failure of Parliament to act. On the other, it is *Parliament's act*.

The final step is

$$-\Phi(x) \leftrightarrow \Phi(-x). \tag{10}$$ [108]

This combines (8) and (9). It states that Parliament has acted and Parliament has not acted. In terms of butterflies, it states that Parliament has not acted with regard to butterflies $-\Phi(x)$ and therefore Parliament has acted to permit the massacre of butterflies. "This conclusion is patently false," Raz writes, supposedly on behalf of Dworkin:

> In England . . . there is no source for the legal proposition that it is legally prohibited to kill any butterfly, but neither is there a source for its contradiction, i.e. that it is not legally prohibited to kill any butterfly. [(7)] must be rejected.[109]

According to Raz, x in (10) stands for "thou shalt not kill butterflies" and -x stands for "you are permitted to kill butterflies." In fact, (10) is the result of switching the meaning of -x between (8) and (9). In (8), -x stands for "I can't find any statute about butterflies" and in (9) -x stands for "there *is* a statute about butterflies." In (8) there is no statute. In (9), there is a statute. In a regime where only parliamentary enactments are law, these are not equivalents. The conclusion in (10), which Parliament's

107. RAZ, AUTHORITY, *supra* note 32, at 55. A "counterposition" or "contrapositive" consists in assuming that the negation of a conclusion produces the negation of the premise. Take the statement "if $x>0$, then $x^3 \neq 0$." The contrapositive asserts, "If $x^3=0$, then $x \not> 0$." JAMES R. MUNKRES, TOPOLOGY 8 (2d ed. 2000).

108. Actually, Dworkin's position is:

$$-x \leftrightarrow -\Phi(x) \text{ or } \Phi(-x).$$

From this it is impossible to derive (10), as Raz has purported to do.

109. RAZ, AUTHORITY, *supra* note 32, at 55.

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failure to enact implies an enactment is the product of surreptitiously switching the meaning of -x.

Although Raz does not say so directly, suppose (10) is a genuine contradiction and not an error in logic. It would be solved by the addition of a moral principle: whatever is not prohibited is permitted. In such a case, Parliament not acting on butterflies is the equivalent of Parliament legislating freedom to kill butterflies. But this addition of a moral principle is not permitted by the precepts of positivism, although Dworkin's jurisprudence would certainly permit it.

Raz needlessly confesses that positivism has been caught dead to rights by the above contradiction (though the contradiction is simply the product of Raz's misunderstanding of Dworkin). He therefore proposes four restrictions to prevent contradiction (in the spirit of Zermelo-Fränkel). None of these is required, as there is no contradiction presented in Dworkin's work (only a tendentious description of positivism). These restrictions are on their own problematic, but as they address a non-existent contradiction, it does not profit us to review them here any further.

CONCLUSION

It is not clear what positivism is supposed to be, but if it asserts that primary rules of law are purely conventional, then positivism gives rise to Russell's Paradox, developed to show that intensive set theory is contradictory. Intension stands for rules of recognition that completely describe the system with no gaps. As a result of set theory, mathematicians now agree that the *practice* of mathematics *exceeds the ability to theorize it*. This is the lesson that jurisprudence must learn. When it does so, it will give up on the quest of eliminating moral criteria from the rule of recognition. Instead, it will find that law is indescribable as an objective matter. We can only tell provisional truths about it. And *that*, if only provisionally, is the truth of jurisprudence.